A SIMPLE METHOD FOR THE ENERGY ESTIMATION OF ELECTRON PAIRS

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ABSTRACT. A simple and practical method is described by which electron pair energies from 2×10^7 ev to 10^{12} ev can be estimated with sufficient reliability. The initial divergence is allowed modification by the multiple coulomb scattering of the electrons, and the energy of the primary photon derived from the observed opening, which is directly measurable. This method when applied to a considerable number of pairs obtained from electromagnetic cascades has been shown to yield meaningful results. The advantages and limitations of the method are discussed.

During the course of an investigation (Kumar, 1956; 1957-a, b; Aditya, 1959a, b) on the phenomena of electromagnetic cascades at high energies, a simple method has been used for estimating energy of electron pairs, and found to yield reliable results. In principle, the initial divergence of the pair and the subsequent multiple scattering of the two partners are both taken account of, so that the energy of the materialising photon can be derived directly from the observed opening of the pair. The influence of multiple scattering on the true opening of a pair has been considered independently also by Lohrmann (1955) who concluded that the observed divergence for pairs of energy $\gtrsim 1$ Bev., is essentially determined by the multiple scattering. Koshiba *et al*, (1954) had also proposed to discuss such an influence of the multiple scattering.

Using the method of energy estimation described below, some results on the mean free path for trident production have been recently published (I), where a brief outline of the method was given. In the present article we propose to discuss the principle alongwith the many approximations and assumptions involved and enumerate the merits, demerits and limitations of the method. The reliability has already been checked (I) by comparing the energies so estimated with those expected from using other methods.

When a photon materialises into a negaton-positon pair, the intermediate angle between the two partners is a function of the energy of the photon and of the ratio of the shared energies. This angle is minimum when the two electrons share the energy equally and increases with the disparity of the pair, the disparity being defined as the ratio of the energy of the low energy electron to the energy

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of the photon. This opening of the pair may be called its true opening and at a certain distance from the origin of the pair, the separation due to it be denoted as d_T.

It is well known that during passage through condensed matter, charged particles undergo multiple coulomb scattering the magnitude of which is a function of the particle momenta. Since the true opening is usually small ($\sim 10^{-3}$ radians for an energy$\sim$a few Bev.,) the multiple scattering is expected to lead almost always to an increase of the true angle. For example, according to Baroni *et al* (1953) the probability that the angle is increased is $\sim \frac{4\pi-\Omega}{4\pi}$ where Ω is the solid angle defined by the aperture of the pair. Let us denote this increase in the separation of the pair by d_S.

Consequently, the observed separation is a resultant of the true opening d_T, and the subsequent separation due to scattering d_S. We shall estimate d_T and d_S in order to find their relative magnitude at various distances from the pair origin.

Stearns (1949) has derived the root mean square value of the angle ω_c between the electron and the direction of the photon, so that in the case of equipartition of energy between the negaton and the positon, the r.m.s., value of the true opening angle of the pair is given as :

$$\omega_p = 2\frac{mc^2}{E} \ln \frac{E}{mc^2} \quad \ldots (1)$$

For the electron rest mass, $m = 0.5$Mev., and photon energy expressed also in Mev., eq., (1) gives

$$\omega_p \simeq 10/E \quad \ldots (2)$$

for photons of energy from 1 Bev., to 100Bev. Following similar arguments, Borsellino (1953) has derived the most probable value of the opening angle, given as

$$\omega_p = \frac{4mc^2}{E} \cdot \phi \quad \ldots (3)$$

where $\phi = 1$ for energy equipartition and ~ 1 even when the energy of one of the electrons is twice than the other. Substitution for m, and for the photon energy in Mev., gives

$$\omega_p \simeq 2/E \quad \ldots (4)$$

The energy found from Stearns' relation (eq. 2) is seen to be about five times that found by using Borsellino's relation (eq.4). Since the latter gives the most probable value, and that in the energy region upto $\sim$ 200 Mev., the results of

Hinterman (1954) suggest better accordance with Borsellino's relation, we have amongst many other workers, preferred to use this relation. Thus

$$d_T \simeq \frac{2}{E} \cdot t \qquad \ldots \quad (5)$$

where t is the distance measured from the pair origin, and has the same units as d_T, say microns.

From the theory of multiple scattering, it is known that when two electrons of equal energy are involved, the mean relative scattering in $t\,\mu$, is given as

$$\bar{\alpha}_{t\mu}{}^0 = \bar{\alpha}_{100\mu}{}^0 \cdot \left(\frac{t}{100}\right)^{\frac{1}{2}} \cdot 2^{\frac{1}{2}} \qquad \ldots \quad (6)$$

where $\bar{\alpha}_{100\mu}$ denotes in degrees the mean scattering angle per 100μ, while $2^{\frac{1}{2}}$ arises on account of the assumed equal scattering of the two electrons. For a photon of energy E (in Mev.,) and equipartition as above,

$$\alpha_{100\mu} = \frac{K}{E/2} = \frac{52}{E} \qquad \ldots \quad (7)$$

where K, the scattering constant is taken = 26 for the units of $\bar{\alpha}$ and E mentioned above. In view of the approximations involved in the method, much purpose is not served by taking into account the variation of the scattering constant with cell size. So that eqs. (6) and (7), with

$$\bar{\alpha}_{t\mu}{}^0 = \frac{ds}{t} \cdot \frac{180}{\pi} \qquad \ldots \quad (8)$$

lead to

$$d_s = \frac{0.128}{E} \cdot t^{3/2} \qquad \ldots \quad (9)$$

where d_S and t, are as usual in microns.

Equating d_T from *eq.*(5) with d_S from eq . (9), it is seen that the contribution due to scattering is as much as that due to mutual divergence at a distance of $\sim$250 μ from the pair origin, while for all larger distances, d_S predominates over d_T. As an illustration let us consider the combined effect of d_T and d_S on a pair of 10 Bev. In figure 1., are plotted the curves between the expected separation against distance from origin. In addition to the curves for the original and scattering corrected separations according to Borsellino's and Stearns' relation, are included two curves, one showing the contribution of multiple scattering alone (curve 3) and another showing the separation expected according to Borsellino's relation for a pair of energy 1 Bev., (curve 6). The close proximity of curve 6, with the other curves for 10 Bev., indicates that without a suitable correction for scattering, the opening angle relations would lead invariably to

an underestimation of energy It is also evident, that as higher energies are approached, it makes little difference as to which one of the Borsellino's or Stearns'

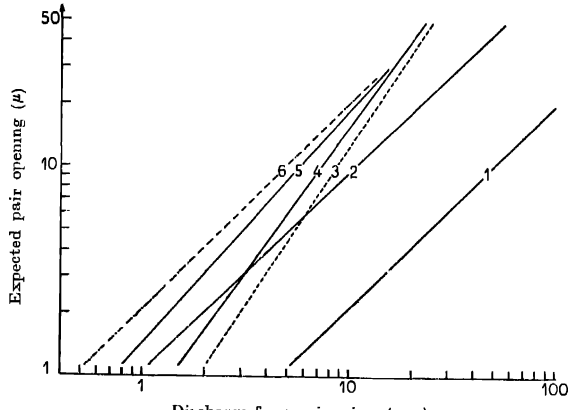

Fig. 1. Expected separation for a pair has been plotted against the distance from pair origin. For a 10 Bev. pair, curves 1 and 2 show the separation according to the relations of Borsellino and Stearns, while curve 3 indicates the root mean square value of the separation due to multiple scattering alone. Consequently, 4 and 5 are the respective modified curves. Curve 6, which gives Borsellino's separation for a 1 Bev., pair is included for comparison.

relation is used or may be that none of the two is essential, as has been concluded by Lohrmann (1955). However, since no sharp cut off can be defined above or below which either of the two contributions due to initial divergence or scattering may be neglected, it appears advantageous to consider at all energies the combined effect of initial divergence and scattering, so that none of the two is deprived of its true importance at various stages of energy and distance from origin.

For various energies from 100 Mev., to 1000 Bev., curves between expected separation and distance from pair origin were drawn in figure 1, of (*I*). These curves can, in short, be expressed in the form of an equation as :

$$E = 6 \cdot d^{-1} \cdot t^{1\cdot4} \qquad \ldots \quad (10)$$

where E is in Bev., t in mm and d in μ.

It had been formerly felt that in those cases, when the energy is so high as to allow no measurement of the separation made within a few *mm* from the origin, the measurements made at larger distances involved uncertainity due to radiation losses, large single scatters and the presence of increasing number of secondary phenomena. At the present stage, most of this difficulty can be overcome by making use of the arguments described very recently by Weill (1959), according

to whom the variation of ionisation along the combined track can be used to derive the separation between the two partners.

The energies of 20 pairs initiating soft cascades, a number of associated pairs in the vicinity of a high energy interaction (Kumar, 1958, Aditya, 1959-c) and the secondary pairs of all these showers have been estimated by this method. Results in the very high region (Table 1, Aditya, 1959a) and the energy spectrum of the secondary electrons (figure 2, Aditya, 1959a) install confidence in the reliability of the energies estimated by this method. In the energy range, where the multiple scattering measurements are meaningful, both these methods yield identical results. There are however two factors that may point out the inaccuracy of the assumptions. Firstly, because of the separation due to relative scattering having an r.m.s., distribution, the most probable value shall not be as much as the r.m.s., separation, so that the method would lead to an overestimation of the pair energy Secondly, since energy equipartition has been assumed, whenever one of the electrons has an energy much different from that of the other, the application of this method is likely to underestimate the energy. It is expected that in most of the cases these two factors might compensate for each other but it cannot be so for all pairs. That is why, for an individual pair the method is not likely to give in all cases the most representative value for the energy. The probability considerations mentioned by Lohrmann (1955) would apply to the distribution of the pair separation as a result of which large discrepancies have to be allowed for in some cases. Inspite of these limitations, the method has a few outstanding advantages. It is perhaps the most simple method and can be applied even to those events which occur in the stack under unfavourable geometrical conditions such as steepness. Unless one needs to go very far from the pair origin, which is not essential in most of the cases, the influence of radiation losses is known to be small. Provided the pair separation is not directly measurable in the vicinity of the origin, it may be derived from the change of ionisation (Weill 1959).

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